# GRAVITOELECTROMAGNETISM <br> AND <br> NEWTON'S LAW OF UNIVERSAL GRAVITATION 

Antoine Acke<br>Retired Professor, Department Industrial Sciences<br>University College Kaho Sint-Lieven<br>Gent - Belgium<br>ant.acke@skynet.be


#### Abstract

Taking into account the kinematics of the gravitating objects, gravitoelectromagnetism (GEM) is a consistent classical field theory about the gravitational phenomena in which context the principle of relativity and the principle of equivalence are valid. In this article it is shown that, in the framework of GEM, Newton's law of universal gravitation perfectly can be deduced and that it can be extended to the case of moving point masses.


Keywords: gravito-electromagnetism, gravitation.

## 1 INTRODUCTION TO GRAVITOELECTROMAGNETISM

In gravito-electromagnetism ${ }^{[1],[2],[3]}$ (GEM) we think about the force acting between two particles $m_{1}$ and $m_{2}$ in terms of "gravitational fields":

1. Particle $m_{1}$ sets up a gravitational field in the space around itself;
2. That field acts on particle $m_{2}$, this shows up in the force that $m_{2}$ experiences.

Generalized: in GEM the gravitational field plays an intermediary role in the interaction between masses.

1. The gravitational field is set up by a given distribution of - whether or not moving particles and it is - just as the electromagnetic field - described as a combination of two threedimensional, time-dependent, intertwined vector fields: the " $g$-field" $\vec{E}_{g}$ and the " $g$ induction" • $\vec{B}_{g}$. These vector fields are, relative to an inertial reference frame $\boldsymbol{O}$, functions of the space and time coordinates. Just like the electromagnetic field ( $\vec{E}, \vec{B}$ ), the gravitational field $\left(\vec{E}_{g}, \vec{B}_{g}\right)$ is mathematically described by a set of four partial differential equations, the "GEM-equations" (or the "Maxwell-Heaviside equations") that describe how $\vec{E}_{g}$ and $\vec{B}_{g}$ vary in space due to their sources - the masses and the mass flows - and how they are intertwined. At a point $P$ of a gravitational field - where $\rho_{G}$ is the mass density and $\vec{J}_{G}$ is the density of the mass flow $-\vec{E}_{g}$ and $\vec{B}_{g}$ must obey the following equations ${ }^{[1]}$ :
[^0]1. $\operatorname{div} \vec{E}_{g}=-\frac{\rho_{G}}{\eta_{0}}$
2. $\operatorname{div} \vec{B}_{g}=0$
3. $\operatorname{rot} \vec{E}_{g}=-\frac{\partial \vec{B}_{g}}{\partial t}$
4. $\operatorname{rot} \vec{B}_{g}=\frac{1}{c^{2}} \frac{\partial \vec{E}_{g}}{\partial t}-v_{0} \cdot \vec{J}_{G}$

And: $\eta_{0} \cdot v_{0}=\frac{1}{c^{2}} \quad$ with $\quad \eta_{0}=\frac{1}{4 \cdot \pi \cdot G}$
Neither these equations nor their solutions indicate an existence of causal links between the " $g$-field" $\vec{E}_{g}$ and the " $g$-induction" $\vec{B}_{g}$. Therefore, we must conclude that a gravitational field is a dual entity having a "field" component and an "induction" component simultaneously created by their common sources: time-variable masses and mass flows.
2. The gravitational interactions can be explained ${ }^{[4]}$ as the effect of the tendency of a material object to accelerate in order to become blind for the gravitational fields generated by other objects. In the context of GEM, the action of the gravitational field on a particle is described by the "force law of GEM", a law analog to Lorentz force law:

A particle, that is moving - relative to an inertial reference frame $\mathbf{O}$ - with velocity $\vec{v}$ in $a$ gravitational field ( $\vec{E}_{g}, \vec{B}_{g}$ ), will accelerate relative to its proper inertial reference frame ${ }^{\bullet} \mathrm{O}^{\prime}$ with an amount $\vec{a}^{\prime}$ :

$$
\vec{a}^{\prime}=\vec{E}_{g}+\left(\vec{v} \times \vec{B}_{g}\right)
$$

We can interpret this by saying that the gravitational field exerts an action on a particle in that field. We call that action the "gravitational force" $\vec{F}_{G}$. It is defined as:

$$
\vec{F}_{G}=m_{0} \cdot \vec{a}^{\prime}=m_{0} \cdot\left[\vec{E}_{g}+\left(\vec{v} \times \vec{B}_{g}\right)\right]
$$

where $m_{0}$ is the rest mass of the particle. From this force law it follows ${ }^{[4]}$ that the effect of the gravitational force on the state of movement of the particle can be expressed by:

$$
\vec{F}_{G}=\frac{d \vec{p}}{d t}
$$

[^1]$\vec{p}=m \cdot \vec{v}$ is the linear momentum of the particle relative to the inertial reference frame $\boldsymbol{O}$. It is the product of its relativistic mass $m=\frac{m_{0}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$ with its velocity $\vec{v}$ in $\boldsymbol{O}$. The linear momentum of a moving particle is a measure for its inertia, for its ability to persist in its dynamic state.

GEM can be considered as an upgrade of Newtonian gravity. Unlike Newton's law of universal gravitation, GEM takes the effect of the kinematics of the gravitating objects into account and it is based on the ideas of motion developed in the context of special relativity. Because the equations of GEM are analogue to Maxwell's equations and because Maxwell's equations are invariant under a Lorentz transformation, the equations of GEM are invariant under a Lorentz transformation so that the Principle of Relativity is valid in the context of GEM. From the postulate of the gravitational action, it follows that he same is true for the Principle of Equivalence.

It is important to state that, in the context of GEM, the "field" is considered as a substantial element of nature and not as a purely mathematical construction. We can the substance of the gravitational field identify as "g-information" ${ }^{[4]}$.

The starting point of GEM differs fundamentally from the starting point of GRT because, in the description by GEM of the gravitational phenomena and laws, space and time don't play an active role. It are elements of the description of nature that do not participate in the physical processes.

It has been shown ${ }^{[7], 8]}$ that certain concrete predictions made on the basis of the gravitoelectromagnetic description of gravity are perfectly in line with the results of cosmological observations.

## 2 NEWTONS UNIVERSAL LAW OF GRAVITATION

The phenomenon of the gravitational interaction between two particles at rest is described by "Newton's universal law of gravitation" ${ }^{[5]}$ :

The force between any two particles having masses $m_{1}$ and $m_{2}$ separated by a distance $R$ is an attraction acting along the line joining the particles and has the magnitude

$$
F=G \cdot \frac{m_{1} \cdot m_{2}}{R^{2}}=\frac{1}{4 \pi \eta_{0}} \cdot \frac{m_{1} \cdot m_{2}}{R^{2}}
$$

where $G=\frac{1}{4 \pi \eta_{0}}$ is a universal constant having the same value for all pairs of particles.
We will show that this law perfectly can be deduced in the framework of GEM.


In fig 1 we consider two particles with rest masses $m_{1}$ and $m_{2}$ anchored at the points $P_{1}$ and $P_{2}$ of an inertial reference frame.

1. $m_{1}$ creates and maintains a gravitational field that at $P_{2}$ is defined by the g-field:

$$
\vec{E}_{g 2}=-\frac{m_{1}}{4 \cdot \pi \cdot \eta_{0} \cdot R^{2}} \cdot \vec{e}_{12}
$$

Indeed. The first GEM-equation - that can be interpreted as the mathematical expression of the conservation of g-information ${ }^{[4]}$ - is equivalent to the statement that the flux of the gravitational field through an arbitrary closed surface $S$ is determined by the enclosed mass $m_{\text {in }}$ according to the law:

$$
\begin{equation*}
\oiint \vec{E}_{g} \cdot \overrightarrow{d S}=-\frac{m_{i n}}{\eta_{0}} \tag{1}
\end{equation*}
$$

Let us apply this equation to an hypothetical sphere $S$ with radius $R$ centered on $P_{1}$.

- Because of the symmetry, $\vec{E}_{g}$ is at every point of that sphere perpendicular to its surface and has the same magnitude. So, at an arbitrary point $P$ of the sphere, $\vec{E}_{g}$ can be expressed as

$$
\vec{E}_{g}=E_{g r} \cdot \vec{e}_{r}
$$

where $\vec{e}_{r}$ and $E_{g r}$ respectively are the unit vector and the component (with constant magnitude) of $\vec{E}_{g}$ in the direction of $\overrightarrow{P_{1} P}$.
Further, at every point of the surface of the sphere: $\overrightarrow{d S}=d S . \bar{e}_{r}$.

With this information we calculate $\oiint_{S} \vec{E} \cdot \overrightarrow{d S}$ :

$$
\begin{equation*}
\oiint_{S} \vec{E}_{g} \cdot \overrightarrow{d S}=\oiint_{S} E_{g r} \cdot \vec{e}_{r} \cdot d S \cdot \vec{e}_{r}=\oiint_{S} E_{g r} \cdot d S=E_{g r} \cdot \not \oiint_{S} d S=E_{g r} \cdot 4 \pi R^{2} \tag{2}
\end{equation*}
$$

- The enclosed mass is $m_{1}$, so

$$
\begin{equation*}
m_{i n}=m_{1} \tag{3}
\end{equation*}
$$

Taking into account (2) and (3), (1) becomes:

$$
\begin{equation*}
E_{g r} \cdot 4 \pi R^{2}=-\frac{m_{1}}{\eta_{0}} \tag{4}
\end{equation*}
$$

We conclude: at a point $P$ at a distance $R$ from $P_{1}$ the gravitational field is pointing to $P_{1}$ and determined by:

$$
\vec{E}_{g}=E_{g r} \cdot \vec{e}_{r}=-\frac{m_{1}}{4 \pi \eta_{0} R^{2}} \cdot \vec{e}_{r}
$$

In particular at the point $P_{2}$ :

$$
\vec{E}_{g 2}=-\frac{m_{1}}{4 \cdot \pi \cdot \eta_{0} \cdot R^{2}} \cdot \vec{e}_{12}
$$

2. If $m_{2}$ was free, according to the postulate of the gravitational interaction it would accelerate with an amount $\vec{a}$ :

$$
\vec{a}=\vec{E}_{g 2}
$$

So the gravitational field of $m_{1}$ exerts a "gravitational force" on $m_{2}$ :

$$
\vec{F}_{12}=m_{2} \cdot \vec{a}=m_{2} \cdot \vec{E}_{g 2}=-\frac{m_{1} \cdot m_{2}}{4 \cdot \pi \cdot \eta_{0} \cdot R^{2}} \cdot \vec{e}_{12}
$$

In a similar manner we find $\vec{F}_{21}: \quad \vec{F}_{21}=-\frac{m_{1} \cdot m_{2}}{4 \cdot \pi \cdot \eta_{0} \cdot R^{2}} \cdot \vec{e}_{21}=-\vec{F}_{12}$

3 THE INTERACTION BETWEEN TWO MOVING POINT MASSES


Fig 2

Two particles with rest masses $m_{1}$ and $m_{2}$ (fig 2 ) are anchored in the inertial frame $\boldsymbol{O}$ ' that is moving relative to the inertial frame $\boldsymbol{O}$ with constant velocity $\vec{v}=v \cdot \vec{e}_{z}$. The distance between the particles is $R$.

Relative to $\mathbf{O}^{\mathbf{\prime}}$ the particles are at rest. According to Newton's law of universal gravitation, they exert on each other equal but opposite forces:

$$
F^{\prime}=F_{12}^{\prime}=F_{21}^{\prime}=G \cdot \frac{m_{\cdot 1} \cdot m_{2}}{R^{2}}=\frac{1}{4 \cdot \pi \cdot \eta_{0}} \cdot \frac{m_{1} \cdot m_{2}}{R^{2}}
$$

Relative to $\mathbf{O}$ both particles are moving with constant speed $v$ in the direction of the $Z$-axis. From the transformation equations between an inertial frame $\boldsymbol{O}$ and another inertial frame $\mathbf{O}^{\prime}$, in which a particle experiencing a force $F^{\prime}$ is instantaneously at rest, we can immediately deduce the force $F$ that the point masses exert on each other in $\boldsymbol{O}^{[6]}$ is:

$$
F=F_{12}=F_{21}=F^{\prime} \cdot \sqrt{1-\left(\frac{v}{c}\right)^{2}}=F^{\prime} \cdot \sqrt{1-\beta^{2}}
$$

We will show that also this form of Newton's law of universal gravitation perfectly can be deduced in the framework of GEM.
1.


At a point $P$ whose position is determined by the time dependent position vector $\vec{r}$ (fig 3) the gravitational field ( $\vec{E}_{g}, \vec{B}_{g}$ ) of a particle with rest mass $m_{0}$ that is moving with constant velocity $\vec{v}=v . \bar{e}_{z}$ along the $Z$-axis of the inertial reference frame $\boldsymbol{O}$ (fig 3 ) is determined by ${ }^{[4]}$ :

$$
\begin{aligned}
& \bar{E}_{g}=-\frac{m_{0}}{4 \pi \eta_{0} r^{3}} \cdot \frac{1-\beta^{2}}{\left(1-\beta^{2} \cdot \sin ^{2} \theta\right)^{\frac{3}{2}}} \cdot \vec{r}=-\frac{m_{0}}{4 \pi \eta_{0} r^{2}} \cdot \frac{1-\beta^{2}}{\left(1-\beta^{2} \cdot \sin ^{2} \theta\right)^{\frac{3}{2}}} \cdot \vec{e}_{r} \\
& \vec{B}_{g}=-\frac{m_{0}}{4 \pi \eta_{0} c^{2} \cdot r^{3}} \cdot \frac{1-\beta^{2}}{\left(1-\beta^{2} \cdot \sin ^{2} \theta\right)^{\frac{3}{2}}} \cdot(\vec{v} \times \vec{r})
\end{aligned}
$$

with $\beta=\frac{v}{c}$, the dimensionless speed of $m_{0}$. One can verify that these expressions satisfy the laws of GEM*.
2. In the inertial frame $\boldsymbol{O}$ of fig 2 , the particles $m_{1}$ and $m_{2}$ are moving in the direction of the $Z$ axis with speed $v . m_{2}$ moves through the gravitational field generated by $m_{1}$, and $m_{1}$ moves through that generated by $m_{2}$.

According the above formulas, the magnitude of the GEM field created and maintained by $m_{1}$ at the position of $m_{2}$ is determined by:

$$
E_{g 2}=\frac{m_{1}}{4 \pi \eta_{0} R^{2}} \cdot \frac{1}{\sqrt{1-\beta^{2}}} \quad \text { and } \quad B_{g 2}=\frac{m_{1}}{4 \pi \eta_{0} R^{2}} \cdot \frac{1}{\sqrt{1-\beta^{2}}} \cdot \frac{v}{c^{2}}
$$

And according to the force law $\vec{F}_{G}=m_{0} \cdot\left[\vec{E}_{g}+\left(\vec{v} \times \vec{B}_{g}\right)\right], \quad F_{12}$, the magnitude of the force exerted by the gravitational field ( $\vec{E}_{g 2}, \vec{B}_{g_{2}}$ ) on $m_{2}$ - this is the attraction force of $m_{1}$ on $m_{2}$ is:

$$
F_{12}=m_{2} \cdot\left(E_{g 2}-v \cdot B_{g 2}\right)
$$

After substitution: $\quad F_{12}=\frac{1}{4 \pi \eta_{0}} \cdot \frac{m_{1} m_{2}}{R^{2}} \cdot \sqrt{1-\beta^{2}}=F_{21}^{\prime} \cdot \sqrt{1-\beta^{2}}$

In the same way we find: $F_{21}=\frac{1}{4 \pi \eta_{0}} \cdot \frac{m_{1} m_{2}}{R^{2}} \cdot \sqrt{1-\beta^{2}}=F_{12}^{\prime} \cdot \sqrt{1-\beta^{2}}$
We conclude that the moving masses attract each other with a force:

$$
F=F_{12}=F_{21}=F^{\prime} \cdot \sqrt{1-\beta^{2}}
$$

This result perfectly agrees with that based on S.R.T.

## 4 CONCLUSION

From the above we can conclude that it's easy to mathematically deduce all aspects of Newton's law from the laws of GEM. The sign of the result indicates that the gravitational interaction between masses is always attractive.

[^2]
## References

1.Heaviside, Oliver. A Gravitational and Electromagnetic Analogy, Part I. sl : The Electrician 1893, Vol. 1893.
2.Jefimenko, Oleg. Causality, Electromagnetic Induction, and Gravitation. sl : Electret Scientific 1992, Vol. 1992
3.Acke, Antoine. Gravitatie en elektromagnetisme. Gent : Uitgeverij Nevelland, 2008.
4.Acke, Antoine. Gravito-electromagnetism explained by the theory of informatons. Hadronic Journal, Vol. 36, Number 4, August 2013 (https://www.researchgate.net/publication/301891607)
5.Resnick, David and Halliday, Robert. Fundamentals of Physics. New York - London - Sydney Toronto : John Wiley \& Sons, 1970.
6. Resnick, Robert. Introduction to special Relativity. New York, London, Sydney : John Wiley \& Sons, Inc, 1968.
7.Arbab I. Arbab. The generalized Newton's law of gravitation versus the general theory of gravity. Journal of Modern Physics, Vol. 3 No. 29, 2012, pp. 1231-1235 (http://arxiv.org/abs/1105.1911v2)
8.Tajmar, M. and de Matos, C.J. Advance of Mercury Perihelion explained by Cogravity. 2003. arXiv: gr-qc/0304104. (http://arxiv.org/abs/gr-qc/0304104)


[^0]:    - also called: gravito-magnetic field

[^1]:    - a reference frame moving relative to $\boldsymbol{O}$ with velocity $\vec{v}$

[^2]:    - Note that $\vec{E}_{g}$ is pointing to the actual position of the particle and not to its light-speed delayed position.

